SHARNIYAH VIJAYAN

B. ENG. (HONS) PETROLEUM ENGINEERING

MAY 2014

SENSITIVITY ANALYSIS OF HYDRAULIC FRACTURE GEOMETRY IN SHALE GAS RESERVOIR

SHARNIYAH A/P VIJAYAN

PETROLEUM ENGINEERING

UNIVERSITI TEKNOLOGI PETRONAS

MAY 2014

SENSITIVITY ANALYSIS OF HYDRAULIC FRACTURE GEOMETRY IN SHALE GAS RESERVOIR

PREPARED BY:
SHARNIYAH A/P VIJAYAN
13801

Dissertation submitted in partial fulfillment of the requirements for the Bachelor of Engineering (Hons)
(Petroleum Engineering)
MAY 2014

Universiti Teknologi PETRONAS
Bandar Seri Iskandar
31750 Tronoh
Perak Darul Ridzuan.

CERTIFICATION OF APPROVAL

Sensitivity Analysis of Hydraulic Fracture Geometry in Shale Gas Reservoir

By

Sharniyah a/p Vijayan

13801

A project dissertation submitted to the

Petroleum Engineering Progromme

Universiti Teknologi PETRONAS

in partial fulfillment of the requirement for the

BACHELOR OF ENGINEERING (Hons)

(PETROLEUM)

Approved by,

____

(Mohammad Amin Shoushtari)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

MAY 2014

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

(SHARNIYAH A/P VIJAYAN)

ABSTRACT

Hydraulic fracturing in shale gas reservoir is a newly developing field. Country like China and USA had encouraged research in developing this unconventional energy field. Shale with unique characteristics such as very low permeability, the existing of microfractures, and sensitivity to contacting fluid make it difficult to evaluate and produce. So it needs an optimum fracturing design to produce but there is absence of proper parameter analysis for hydraulic fracture geometry is captured in the literature.

This study is carried out to identify the best 2-dimensional fracture propagation model of hydraulic fracturing in shale gas reservoir and to evaluate the response of models by controlling the parameters such as viscosity of fracturing fluid, injection rate and injection time. Meanwhile the study will be based on all the research papers, journal and books. Software like MATLAB will be used to develop the mathematical code and the parameters will be analyzed using the code.

Besides that, 2-dimensional models will be list out through studies. Then, the best model will be chosen and mathematical code is developed. From the code, the effect of manipulating the parameters on the outcomes such as average width, fracture length, wellbore width and wellbore net pressure will be observed and analysed. This is to verify the use of 2D fracture propagation model in the hydraulic fracture geometry in shale gas reservoir.

ACKNOWLEDGEMENT

In completion of this final year project, I would like to thank Universiti Teknologi Petronas for providing me the opportunity to conduct this study. I would also like to express my gratitude to my supervisor, Mr Mohammad Amin Shoushtari for his unconditional guide and support throughout the project. Finally, I would like to thank my parents and friends for their continuous support and encouragement which driven me to complete this project.

Table of Contents

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

h_f	Fracture height, m
$W_{w,o}$	Wellbore width, m
μ	Viscosity, Pa
x_f	Fracture length, m
E’	Plane strain modulus, Pa
$\overline{\mathrm{w}}$	Average width, m
S_p	Spurt loss coefficient, m
C_L	Leak off coefficient, $\mathrm{m.s}^{-1/2}$
B	Auxiliary variable for carter equation II, dimensionless
t	Time, s
P	Wellbore net pressure
E	Young’s modulus
V	Poisson’s ratio

CHAPTER 1: INTRODUCTION

1.1 BACKGROUND

This project is about analysis of parameters that influence hydraulic fracture geometry applied in shale gas reservoirs. Hydraulic fracturing treatment has long been applied to stimulate oil and gas reservoirs. Meanwhile it is new technique been used in shale gas reservoirs. Hydraulic fracturing technique is defined as pumping high viscous fluids at a sufficiently high pressure into the completion interval until fracture is formed.

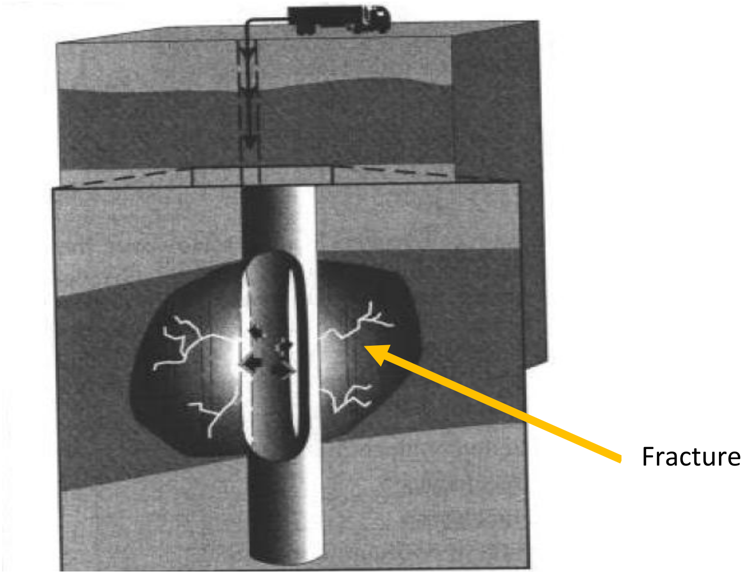


Figure 1: Internal pressure breaking a vertical wellbore

This fracture is then filled with high conductivity proppant which hold the fracture open after the treatment is finished. The proppant will be transported down to the fracture with fracturing fluid. The fracturing fluid used can be in form of foam, gel or slickwater based depending on the composition needed for the fracturing.

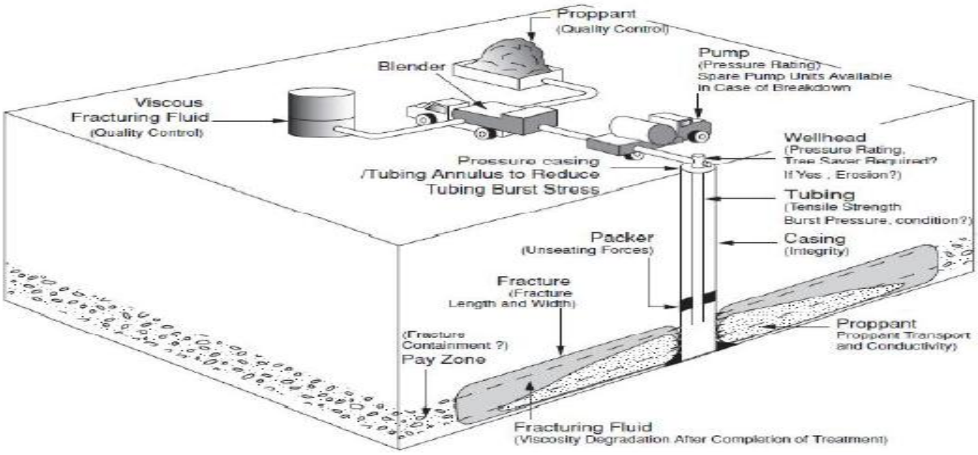


Figure 2: Propped hydraulic fracturing treatment

Shale is fine sedimentary rocks that are rich in source of petroleum and natural gas. It is made up of clay size weathering debris. Recently, the use of hydraulic fracturing treatment method has allowed and increased access to large volumes of shale gas that were previously uneconomical to produce. From a reservoir development point of view, having a reasonable understanding of hydraulic fracture geometry is crucial for determining well spacing and for devising field development strategies design to produce more hydrocarbons (Bennett et.al, 2005).

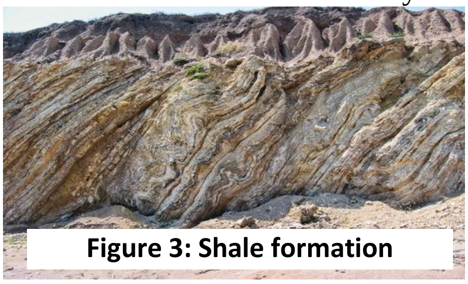


Figure 3: Shale formation

Besides that, shale is categories by its characteristics which parallel with the bedding, called fissility, and will split along these planes. Characteristic of split shale is flat flakes, thin and soft enough to be broken by hand (Merriman, Highley and Cameron, 2003).

Figure 4: Shale split into thin flakes along the bedding fissility

Thus, to understand the hydraulic fracture geometry study about fracture propagation model is important. The purpose of this study is to find best 2-dimensional model and manipulate the parameters involved to produce best outcomes with available shale rock mechanical properties and data. Simplified geometry like 2-dimensional models is often tractable mathematically. There are 2 major models that is Perkins, Kern & Nordgren (PKN) model, and the Khristianovich & Zheltov, Geertsma & deKlerk (KGD) is studied. For given shale gas reservoir, one must choose a set of values for different parameters such that good hydraulic fracture geometry or model is used to develop the design. There are 3 parameters that will be analyse here 1) viscosity of fracturing fluid, 2) injection rate of the fluid, and 3) injection time. Mathematical code developed based on selected model will be used to analyse the parameters with different value. All the information will be collected by research studies.

1.1 PROBLEM STATEMENT

Shale with unique characteristics such as very low permeability, the existing of microfractures, and sensitivity to contacting fluid make it difficult to evaluate and produce. The rock strata in which shale gas is trapped are almost impermeable to gas flow. Therefore it is necessary to do hydraulic fracturing to open the tiny pores where the gas is held. On the other hand, it is a developing field, so absence of proper parameter analysis for hydraulic fracture geometry is captured in the literature.

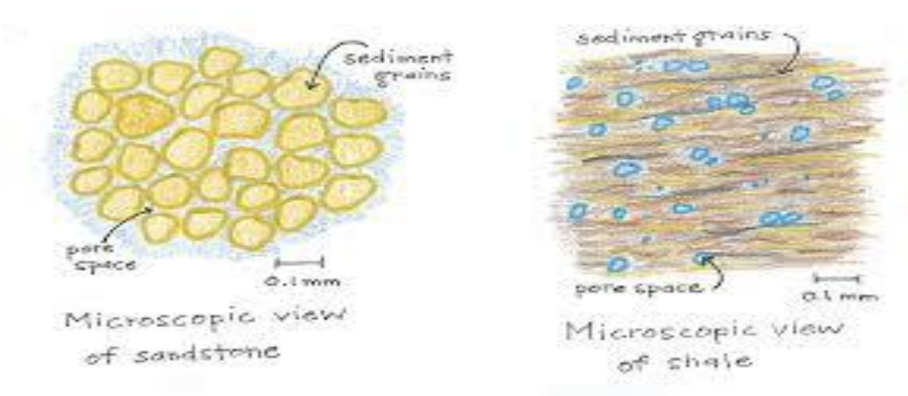


Figure 5: Comparison between sandstone and shale rock characteristics

Thus to achieve the optimum design and high production, selection of correct fracture geometry and analysis of parameters is important. From literature study, effective 2- dimensional model and its parameters are studied to evaluate the shale formation for better hydraulic fracture geometry.

1.2 OBJECTIVE

The aim of the project is:

- To identify the best 2-dimensional fracture propagation model to be used in shale gas reservoir.
- To evaluate the response of models by controlling the parameters.
    - Viscosity of fracturing fluid
    - Injection rate
    - Injection time

1.3 SCOPE OF STUDY

The study will be based development of shale gas reservoir using hydraulic fracturing method. Besides that, the study will be also based on 2-dimansional fracture propagation models that help in hydraulic fracturing design. The selected model will be developed in mathematical code using MATLAB software. Analysis of the mathematical code using different parameters that influence hydraulic fracture geometry is carried out.

1.4 RELEVANCY OF THE PROJECT

- *Petroleum engineering*
  1) Development and usage of mathematical model requires application of knowledge from computational method course.
  2) Understanding of hydraulic fracturing and shale gas reservoir requires knowledge from courses such as well stimulation technique and principle of reservoir engineering.
- *Shale gas reservoir* – The production of hydrocarbon from conventional reservoir might become insufficient for consumers. Shale is newly developed field for hydrocarbon production and many countries had invested in the research of shale reservoir development.

1.5 FEASILIBILITY OF THE PROJECT WITHIN THE SCOPE AND TIME FRAME

- *Scope of study* - This project was carried out within the scope of petroleum engineering course as it encompasses various aspects of this field of study.
- *Time allocation (2 semesters)* - The time frame is sufficient for a complete study on the literatures available on this topic as well as to analysis the parameters that influence the hydraulic fracture geometry and relate the model to shale gas reservoir.

CHAPTER 2: LITERATURE REVIEW OR THEORY

2.1 2-DIMENSIONAL MODEL

2D model consider vertical hydraulic fractures, where any horizontal geometry cross-section and the processes in it do not depend on height. The major advantage of two-dimensional (2D) compared to 1D model is that 2D models give the chance to describe effects interconnected to the fracture curvature around the wellbore (Cherny, Chirkov, Lapin & Muranov, 2009). 2D model include Perkins-Kern-Nordgren (PKN) fracture model, and Khristianovich-Geertsma-de. Klerk (KGD) fracture model.

2.2 FRACTURE GEOMETRY

2.2.1 Perkins, Kern & Nordgren (PKN) Geometry

Perkins and Kern in 1961, developed equations to compute fracture length and width with a fixed height. They modified the classic Sneddon plane strain crack solution to expand the PK model. After that, Nordgren in 1972 improved this model by adding up fluid loss to the solution, hence, this model is commonly called PKN model (Valko et.al, 1995). The PKN model assumes that fracture toughness could be neglected, because (Xiang, 2011):

- Energy required for fracture to propagate was significantly less than that required for fluid to flow along fracture length.
- Plane strain behavior in the vertical direction,
- Fracture has a constant height, and propagates along the horizontal direction

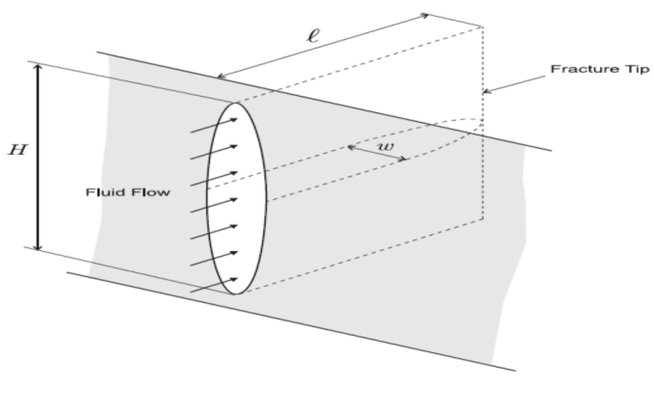


Figure 6: Schematic diagram of PKN fracture geometry

The PKN geometry is of an elliptical shape in both the vertical and horizontal axes (Valko & Economides, 1995). The height is constant and the length is considerably larger. Meanwhile, the fluid pressure is assumed to be constant in each vertical cross section perpendicular to the direction of propagation. The PKN model is applicable to long fractures of restricted height and elliptical vertical cross section.

2.2.2 Khristianovich & Zheltov, Geertsma & deKlerk (KGD) Geometry

KGD model was developed by Khristianovitch and Zheltov (Khristianovitch andZheltov 1955) and Geertsma and de Klerk (Geertsma and Klerk 1969). It considers fracture mechanics effects on the fracture tip, and simplifies the solution by assuming that the flow rate and pressure in the fracture is constant along the majority of the fracture length, except for a small region close to the tips. The KGD model for width calculation does not depend on height, and is used for short fractures where plane strain assumptions are applicable to horizontal sections (Adachi, Siebrits, Peirce & Desroches, 2007). This is only applicable if fracture length is much smaller than fracture height.

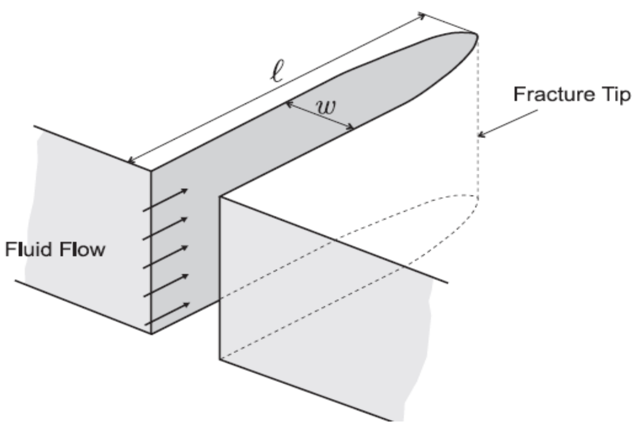


Figure 7: Schematic drawing of KGD fracture geometry

In conclusion, KGD model has six assumptions (Geerstma & Klerk, 1969):

- the fracture has an elliptical cross section in the horizontal plane
- each horizontal plane deforms separately
- fracture height, h_f, is steady
- fluid pressure in the propagation direction is determined by flow resistance in a narrow rectangular, vertical slit of variable width
- fluid does not flow through the entire fracture length
- cross sections in the vertical plane are rectangular (fracture width is constant along its height)

2.3 PARAMETERS IN TREATMENT

2.3.1 Viscosity of fracturing fluid

Shale formation is a formation with existence breaks and microfractures. This is why a low viscosity fluid can penetrate more easily the microfractures and break, and transmit the injection pressure to all the penetrated points. It makes the shale cores become much weaker and allow the low viscosity fluids to break it at low pressures (Gomaa, Qi Qu, Maharidge, Nelson & Reed, 2014). On the other hand, the high viscosity fluids need high pressure to penetrate in the microfractures and breaks. The fluids tend to remain within the hole in low pressure. Fluid viscosity determines the fracture length, fracture maximum width, fracture geometry changes and fracturing pressures (Xiang, 2011).

The fluid diffusion using high viscosity fluids reduce fracture complexity, as evidenced by:

- less complex micro seismic pattern that is developed,
- smaller penetration distance
- create fewer secondary fractures.

2.3.2 Injection rate

Increasing in injection rate of fluid will decreases the breakdown pressure needed to fracture shale formation. Besides that, the injection rate is proportional to the build up rate. When we increase the injection rate, the build up rate also will increase. This will allow more fluid to enter the shale pore at higher injection rate and transmit the injection pressure into more points inside the shale fracture, thus reduce the breakdown pressure (Gomaa, Qi Qu, Maharidge, Nelson & Reed, 2014).

On the other, very high injection rate also can cause problem. Higher injection rate will increase treating and surface pressure. Higher treating pressure may go above the formation critical pressure, therefore induce uneconomic fracture growth while higher surface pressure may spoil surface equipment (Rahman, 2001).

2.3.3 Injection time

When come to optimum injection rate, there are always an optimum injection time which is necessary to develop the most favourable fracture size. Any prolonged injection after the optimum time will create unnecessary fracture development and thus acquire additional treatment cost (Rahman, 2001).

2.4 SHALE GAS RESERVOIR

Shale gas reservoir refers to natural gas that is trapped under shale formation. Based on geologists it is known that the gas is held in the shale not only in small pores, but also in a solid solution bound onto the rock grains. Shale gas has emerged as one of the energy source since developed the Mississippian Barnett Shale in the Fort Worth Basin with application of horizontal drilling and hydraulic fracturing (Chopra et.al, 2012).

This initiated US geologists to look for more shale basins in US which resulted in the finding of the Devonian Antrim shale of the Michigan Basin, the Devonian Ohio Shale of the Appalachian Basin, the Devonian New Albany Shale in the Illinois Basin and the Cretaceous Lewis Shale in San Juan Basin. Followed by, development of the Fayetteville Shale in Arkansas and the Woodford Shale in Oklahoma in 2004 and Haynesville Shale in 2008 (Chopra et.al, 2012).

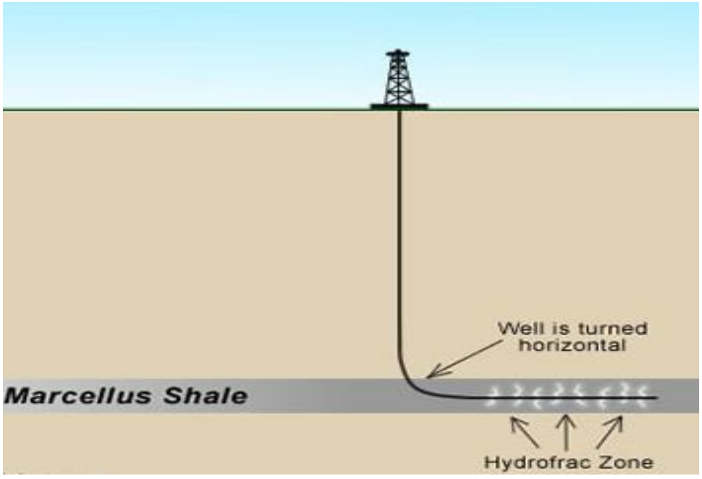


Figure 8: Concept of a horizontal well with hydraulic fracturing zone

Matrix porosity and permeability is low in shale gas reservoir so it relies on natural fractures. When there is no natural fracture, the reservoir is stimulated using hydraulic fracturing. Besides that, shale gas reservoir has low recovery factor (20%) compared to conventional reservoirs. Chopra said that permeability path only can be created in shale through natural fractures. In shale formation, natural fractures can be detected by azimuth variations of the velocity examination (Treadgold et.al, 2011).

2.4.1 Numerical modelling and studies in shale gas reservoir

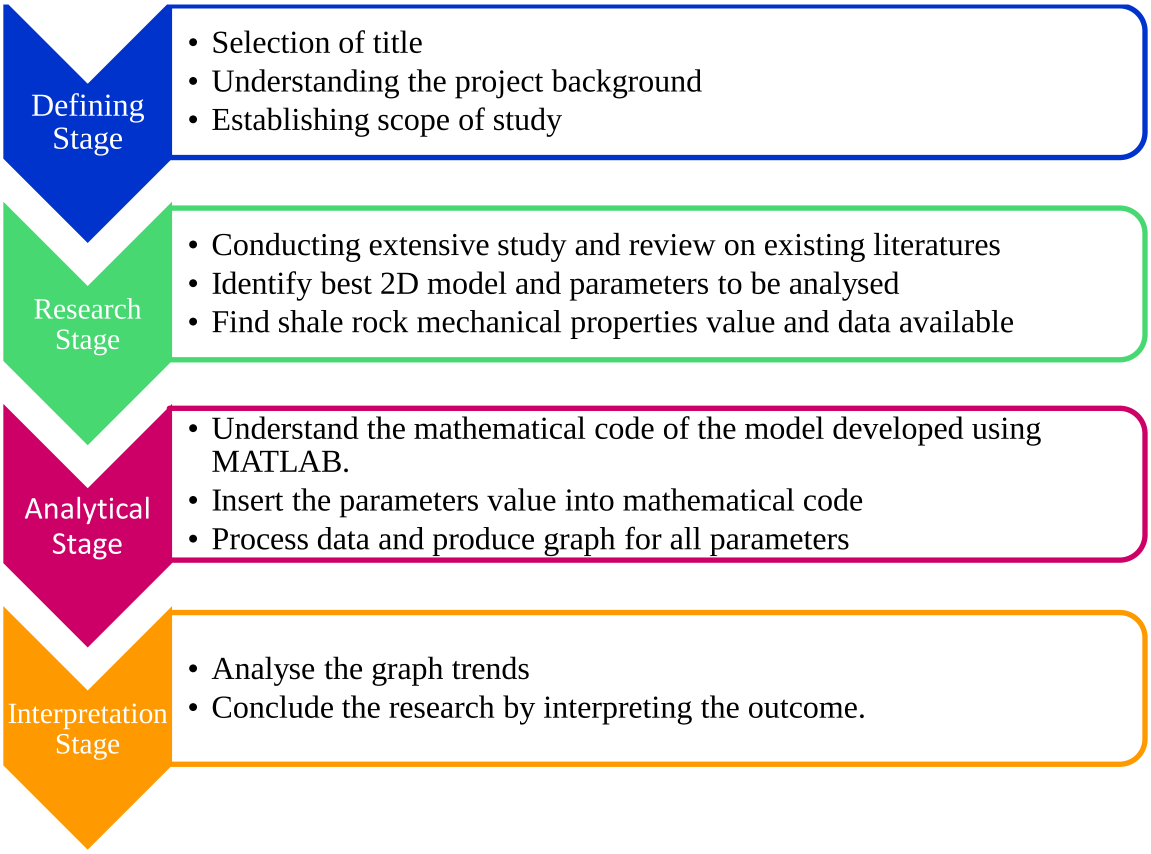
Several analytical models and numerical studies have been conducted to predict flow performance, evaluate the impact of different reservoir and fracture properties on production. Nolte in 1979 has done a research about parameters that quantify a fracture and fracturing process which can be determined fracturing pressure decline. The outcome of the investigation is from the pressure decline data after a treatment, the parameters which illustrate the geometry can be obtained directly. Besides that, the fluid efficiency and fracture closure time can be estimated from the decline pressure ratio (Nolte, 1979).

In 1980, the study about an analysis of hydraulic fracture containment effects has been carried out. It is proven that stress intensive factor and fracture toughness have only partial applicability in hydraulic fracturing Hence, in normal treatment, the fracture will penetrate into the layers next to the pay zone that is being fractured. However, the effects of contrasts in stiffness and in-situ stress between pay zone and adjoining layers will limit the penetration depth of the fracture into layers (Eekelen, 1980).

Next, in 2011 the complex geologic domains and fractures in three dimensions are determined using a grid construction tool to generate high-resolution unstructured meshes using Voronoi grids. This grids help in evaluation of interaction between propped hydraulic fractures and secondary “stress-release” fracture (Olorode, 2011). From the study outcome, the production signature and flow regime can be identified.

CHAPTER 3: METHODOLOGY

3.1 PROJECT FLOW CHART



3.2 GANTT CHART AND KEY MILESTONE

3.2.1 FYP I

No	Detail/ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Selection of project topic														
2	Preliminary research work • Find journals and books related to topic • Understand the basic concept of 2D models • Collect data about the parameters influenced the model														
3	Submission of extended proposal • Complete literature review and data						○								
4	Proposal defence														
5	Project work continues • Analyse the models and the parameters • Choose the model with justification														
6	Submission of Interim draft report • Add summary of progress under result and discussion part													○	
7	Submission of interim report														○

Process

○ Suggested milestone

3.2.2 FYP II

No	Detail/ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Project work continues - Find shale rock mechanical properties - Collect data for parameters - Test the outcome of the code with example parameters														
2	Submission of progress report - Amend with work progress														
3	Project works continue														
4	Pre- SEDEX														
5	Submission of draft final report														
6	Submission of dissertation (soft bound)														
7	Submission of technical paper														
8	Viva														
9	Submission of project dissertation (hard bound)														

Process

Suggested milestone

3.3 RESEARCH METHODOLOGY

- Through research studies, find 2-dimensional fracture propagation models used for hydraulic fracturing to find hydraulic fracture geometry.
- Screen all the models and find the best model based on justification.
- Develop a mathematical code using MATLAB software. (Algebraic expression).
- Shale rock mechanical properties values are used to relate the model to shale rock.
- Relevant parameters value collected from literature and manipulated to set as input for the mathematical code. The input value is inserted into the code. Then, the data is processed and calculated.
- Graph of each parameter against each fracture geometry variable is produced. The effect of parameters on the fracture geometry is analyzed based on the graph trend.
- The application of 2D fracture propagation model in shale gas reservoir is verified.

CHAPTER 4: RESULT AND DISCUSSION

4.1 SELECTED 2D FRACTURE PROPAGATION MODEL

From all the models, PKN-C model is selected as best model to be used for fracture geometry determination in shale gas reservoir. We can determine the wellbore width, average width, fracture length, and pressure from this mathematical model. The mathematical model is shown below.

- **Wellbore width**

$$\mathbf{W_{w,0} = 3.27\,(\frac{\mu i x_f}{E'})^{1/4}}$$

-$W_{w,0}$: Wellbore width, m
-μ : Viscosity of the fluid, Pa.s
-I : Injection rate, $m^3.s^{-1}$
-x_f : Fracture length, m
-E' : Plane strain modulus, Pa

- **Average width**

$$\mathbf{\bar{w} = \frac{\pi}{5} w_{w,0} = 2.05\,(\frac{\mu i\, x_f}{E'})^{1/4}}$$

-$\bar{w}$: Average width, m
-μ : Viscosity of the fluid, Pa.s
-x_f : Fracture length, m
-i : Injection rate, $m^3.s^{-1}$
-E' : Plane strain modulus, Pa

- **Fracture length**

$$\mathbf{x_f = \frac{(\bar{w} + 2S_p)i}{4C_L^2\pi h_f}\,[\exp(\beta^2)\,\mathrm{erfc}(\beta) + \frac{2\beta}{\sqrt{\pi}} - 1;} \text{ where } \mathbf{\beta = \frac{2C_L\sqrt{\pi t}}{\bar{w}+2S_p}}$$

-x_f : Fracture length, m
-$\bar{w}$: Average width, m
-S_p : Spurt loss coefficient, m
- i : Injection rate,$m^3.s^{-1}$
-C_L : Leak off coefficient, $m.\,s^{-1/2}$
-h_f : Constant fracture height, m
-β : Auxiliary variable for Carter equation II, dimensionless
- t : Time, s

- **Pressure**

$$\mathbf{p_{n,w}} = \frac{\mathbf{E'}}{\mathbf{2h_f}} \mathbf{w_{w,0}}$$

-$p_{n,w}$: Wellbore net pressure, Pa
-E' : Plane strain modulus, Pa
-h_f : Constant fracture height, m
-$W_{w,0}$: Wellbore width, m

4.1.2 Justification for the selection

Based on study of all the tabulated models, PKN-C is chosen as best model because:

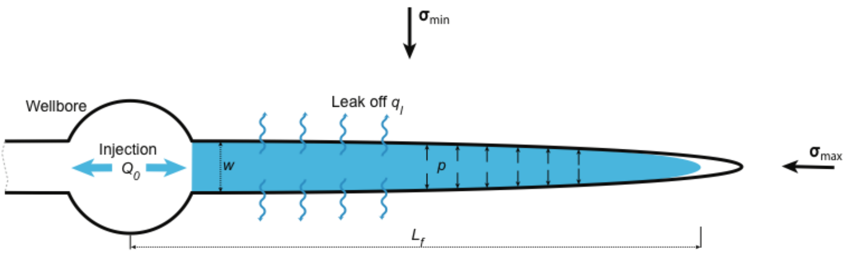


Figure 9: Schematic diagram of fluid leakoff

- **Fluid leakoff is considered in the equation that is by Carter equation II**

The alphabet C in PKN-C stands for Carter. In Carter equation, material balance is formulated in terms of flow rates. The time, t the injection rate entering one wing of the fracture, should be equal to the sum of the different leakoff rates plus the growth rate of the fracture volume

- **It can use for both Newtonian and non-Newtonian fluid**

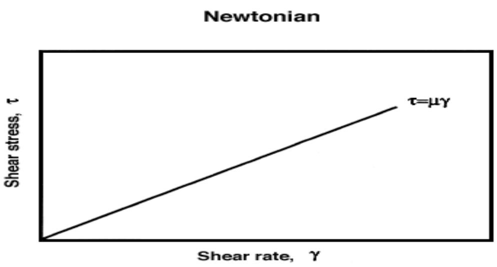


Figure 10: Newtonian fluid characteristics

Newtonian fluid is the fluid that only required viscosity as material function to calculate pressure drop and flow rate. The viscosity is independent of a shear rate while it exhibit direct proportionality between shear stress and shear rate.

Non-Newtonian fluid is a general model in which the viscosity depends on shear rate such as power law model. It is a non linear relationship between shear stress and shear rate, which is more descriptive of real fluid behavior.

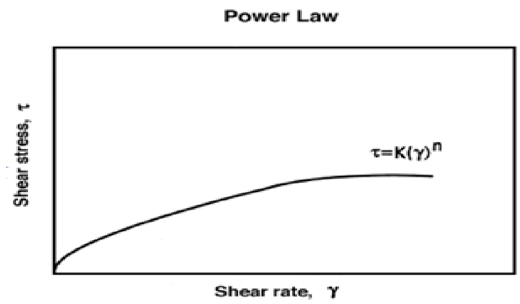


Figure 11: Non-newtonian fluid characteristic

- **It predicts fracture lengths closer to the computed by 3D model**

All the 2D fracture propagation types and mathematical code developed in MATLAB is attached in the appendix.

4.2 ANALYSIS OF THE OUTCOME

The plane strain modulus, E' of the shale rock is calculated using mechanical rock properties. The formula and value used is shown below:

$$\boldsymbol{E' = E/(1 - v^2)}$$

E- Young's Modulus, Pa
v- Poisson's ratio

Young's modulus range: **0.05-0.59**
Poisson's ratio: **0.1-0.43**

In the calculation maximum young's modulus value (0.59) and average Poisson's ratio is used (0.25). The calculated value of plane strain modulus is 6.29x 10^10. All the data used to find different viscosity effect of fluid is tabulated below.

The response of the model with different viscosity of fracturing fluid, injection rate and injection time is produced using graphs. The trend of the graph is analysed and the reason of such outcome is also studied. Thus, the result and analysis of various parameters are discussed below.

4.2.1 Viscosity of fracturing fluid

The input values insert into the mathematical code is tabulated below.

Leak off Coefficient, 〚m/s〛^(1/2)	Spurt loss, m	Height, m	Plane strain modulus, Pa	Viscosity, Pa.s	Injection rate,m^3/s	Time,s
0.00000984	0	51.8	62933333330	0.05	0.025	12 000
0.00000984	0	51.8	62933333330	0.1	0.025	12 000
0.00000984	0	51.8	62933333330	0.3	0.025	12 000
0.00000984	0	51.8	62933333330	0.5	0.025	12 000
0.00000984	0	51.8	62933333330	0.6	0.025	12 000
0.00000984	0	51.8	62933333330	0.8	0.025	12 000

Table 1: Input value of viscosity of fluid

Graphs below show the effect of viscosity of fracturing fluid on average width, fracture length, wellbore width and pressure.

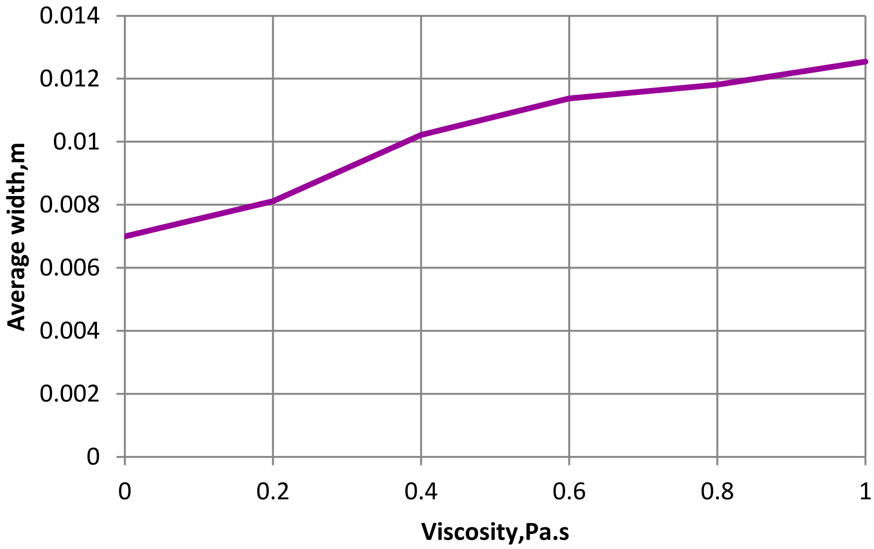


Figure 12: Effect of different viscosity of fluid on average width

Refer to graph above, we know that the average width of fracture increases when viscosity of fracturing fluid increases. From 0Pa.s to 0.2Pa.s, the average width also increases slightly from 0.007m to 0.008m approximately and from 0.2Pa.s to 0.4Pa.s, the average width increases from 0.008m to 0.01m. The average width is 0.0117m to 0.012m from 0.6Pa.s to 0.8Pa.s viscosity and there are only slight changes in average width from 0.8Pa.s to 1.0Pa.s viscosity that is from 0.0119Pa.s to 0.0122Pa.s. The average width will be small along the fracture if less viscous fluid is used for fracturing.

Increase in viscosity will decrease the mobility of the fracturing fluid. At constant injection rate, using less viscous fluid will reduce the average width. Thus, more viscous fluid is required to transport the proppant down to the fracture while break the formation and increase the width of the fracture. This graph will help to choose fluid with right viscosity to provide sufficient fracture width to ensure proppant enter into the fracture .

Graph below show the effect of different viscosity on fracture length. The fracture length decreases as viscosity increases. From 0Pa.s to 0.2pa.s the fracture length decreases from 580m to 520m and from 0.2Pa.s to 0.4Pa.s, the fracture length decreases from 520m to 440m. On the other hand, the fracture length decreases linearly from 0.6Pa.s to 1.0Pa.s. There is only slight change in fracture length from 0.6Pa.s to 1.0Pa.s viscosity.

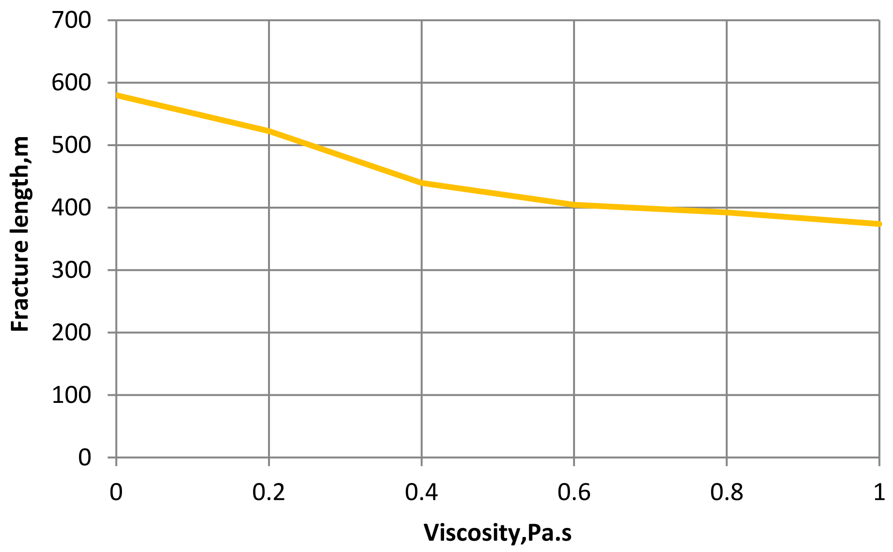


Figure 13: Effect of different viscosity of fluid on fracture length

High fluid viscosity is required to transport proppant down a long fracture. However, high viscosity increases the net pressure inside the fracture. This result in stress difference between the overlying and underlying shale and causes height growth, make less penetration than needed, thus less viscous fluid is required to transport the proppant and increase the fracture length. Besides that, the constant injection rate used also decreases the fracture length as viscosity increases.

Graph below show the effect on wellbore width with different viscosity of fracturing fluid. The graph shows that the wellbore width increases when viscosity of fracturing fluid increases. From 0Pa.s to 0.2Pa.s, the average width increases slightly from 0.012m to 0.013m approximately and from 0.2Pa.s to 0.4Pa.s, the average width increases from 0.013m to 0.016m. The average width is 0.0016m to 0.00175 at from 0.6Pa.s to 0.8Pa.s viscous fluid and 0.0175m to 0.019m from 0.6Pa.s to 0.8Pa.s in viscosity. From 0.8Pa.s to 1Pa.s there is no significant increase in the viscosity before ending at 0.02. The wellbore width is larger if we used viscous fluid for fracturing.

Viscous fluid is needed to transport proppant from wellbore to tip of fracture. Wellbore width also increases when viscosity of fracturing fluid increases because viscous fluid helps to overcome near wellbore effect such as tortuosity and formation damage. However, there is an only slight increase in wellbore width as viscosity increases because the injection rate and time is constant in this case.

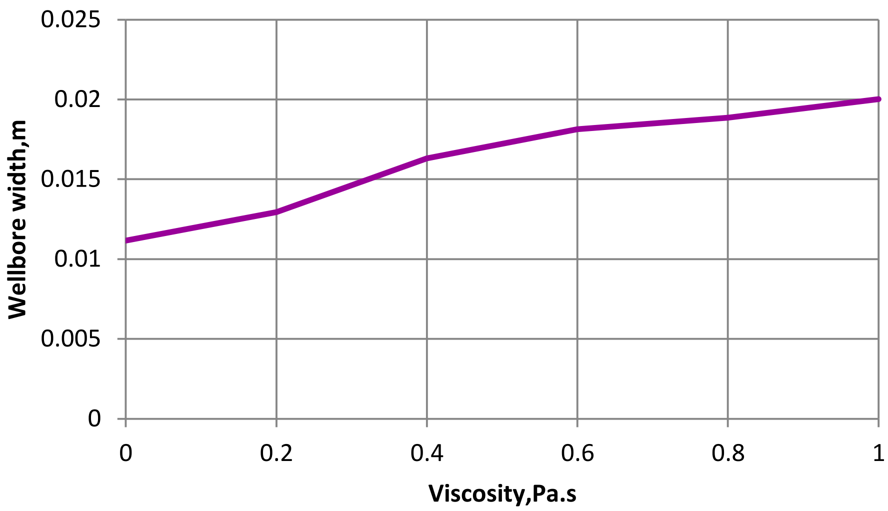


Figure 14: Effect of viscosity of fluid on wellbore width

The graph below shows the effect of different viscosity of fracturing fluid on wellbore net pressure. As viscosity of fluid increases the pressure required to transport it also increases. The initial pressure required for 0.2Pa.s viscous fluid is 650kPa pressure. Then, the pressure increases to 850kPa for 0.4Pa.s viscous fluids and 950kPa for 0.6Pa.s viscous fluid. There is only slow rises in pressure from 0.6Pa.s to 1.0Pa.s that is 950kPa to 1000kPa approximately. The pressure goes up to 950kPa to 1000kPa from 0.6Pa.s to 0.8Pa.s in viscosity. And from 0.8Pa.s to 1Pa.s there is no significant increase in the pressure before it settles slightly higher than 1000kPa.The pressure increases gradually from at 0.2Pa.s to 0.4Pa.s viscous fluid that is 200kPa.

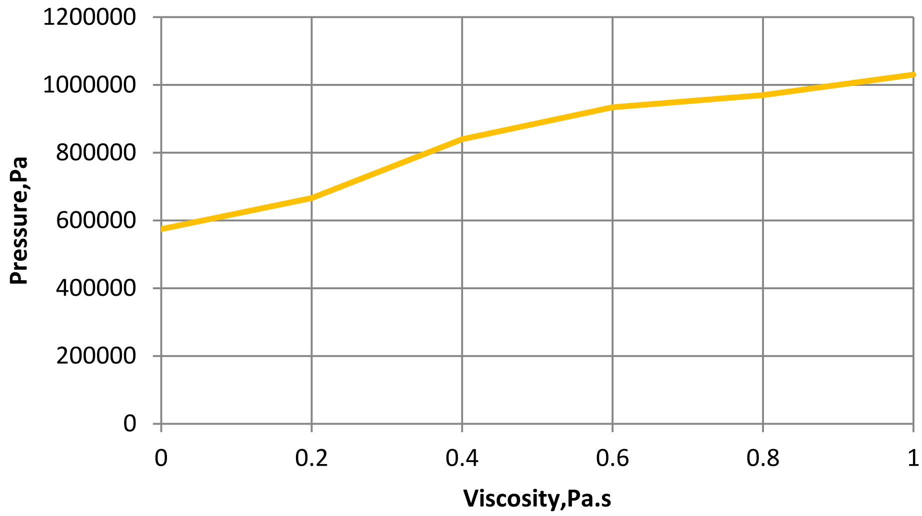


Figure 15: Effect of different viscosity of fluid on wellbore net pressure

The pressure increases as viscosity increases because the mobility or flow capability of fluid will reduce as viscosity of the fluid increases. Due to friction effect within the fracture, the pressure is not stable over the entire fracture. Thus, more pressure is needed to transport the fluid along the fracture. The fracture initiation pressure should be higher than fracture propagation pressure. This shows that high viscous fluid need high pressure to initiate the flow of the fluid.

In conclusion, we need to use viscous fluid to transport high proppant concentration down to the fracture whish also increase the average and wellbore width. However, increasing viscosity will decrease the mobility of fluid which reduce the fracture length so high injection rate need to use to transport the fluid down to the fracture until the fracture tip.

4.2.2 Injection rate of fluid

The input values insert into mathematical code for injection rate is tabulated below.

Leak off Coefficient, 〚m/s〛^(1/2)	Spurt loss, m	Height, m	Plane strain modulus, Pa	Viscosity, Pa.s	Injection rate,m^3/s	Time,s
0.00000984	0	51.8	62933333330	0.2	0.0005	12000
0.00000984	0	51.8	62933333330	0.2	0.003	12000
0.00000984	0	51.8	62933333330	0.2	0.01	12000
0.00000984	0	51.8	62933333330	0.2	0.025	12000
0.00000984	0	51.8	62933333330	0.2	0.04	12000
0.00000984	0	51.8	62933333330	0.2	0.066	12000

Table 2: Input value of injection rate

Graphs below show the effect of viscosity of fracturing fluid on average width, fracture length, wellbore width and pressure. Refer to graph below, it is known that the average width increases when injection rate of fluid increases. It can be said that injection rate is almost directly proportional to the average width. At the initial stage of the injection, the width expands about 0.001 to 0.002m and the plot clearly shows that the width gets wider as the injection rate increases. Finally, at 0.01 cubic m per second the average width is 0.0085m.

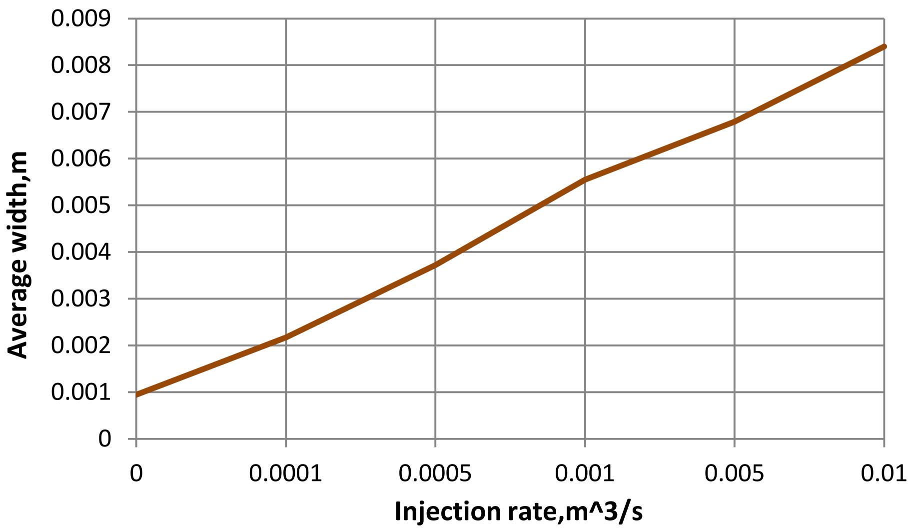


Figure 16: Effect of injection rate on average width

Based on the trend of the line we know that the width along the fracture increases as injection rate increases. This is because increasing in injection rate will increase the flow of fracturing fluid with proppant down to the fracture. Besides that, increase in proppant concentration will increase the width. Based on study, it is known that higher injection rate will give larger fracture with greater width but it not yield most efficient.

Refer to graph below, it is known that the fracture length increases when injection rate of fluid increases. At the initial stage of injection where 0 to 0.0001 cubic metres per second injection rate has been used, the fracture length tends to be elongate from 0 to 100m. At injection of 0.0005 cubic metres per second the fracture length is 300m. There is a linearly increment from 0.0005 to 0.005m and the fracture length is approximately 1000m at 0.005. Finally when the injection rate, is about 0.01 cubic metres per second the length of fracture has been recorded as 1300m.

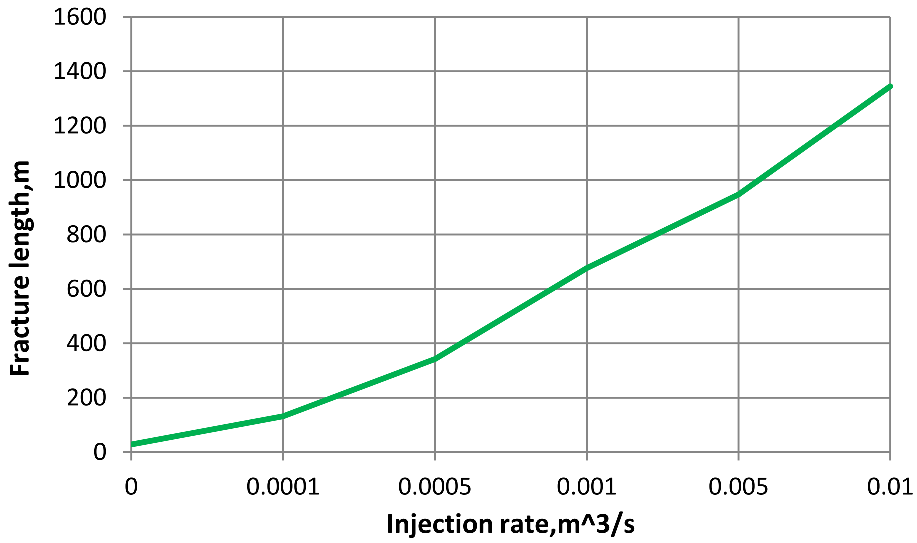


Figure 17: Effect of injection rate on fracture length

The fracture length increases gradually as injection rate increases because injection rate is important variables in hydraulic fracturing. Injection rate is the property that can control the distance of flow of fracturing fluid and proppant. With constant viscosity of fracturing fluid and type, by increasing the injection rate, the length of the fracture also can be increase

The graph below shows the effect of injection rate on wellbore width at fracture.

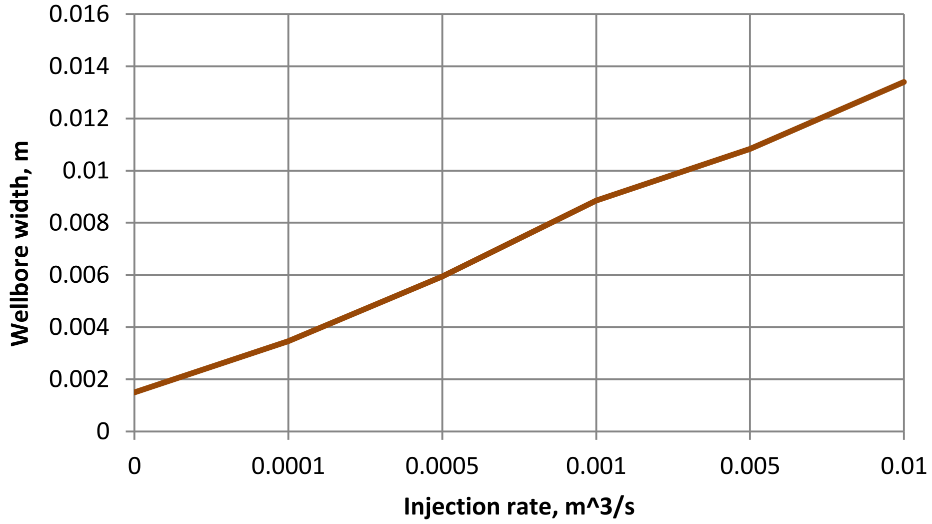


Figure 18: Effect of injection rate on wellbore width

From the graph above, it is proved that wellbore width increases as injection rate increases with other values keep constant. The injection rate rises from 0 to 0.001 cubic metres per second with wellbore width of 0.0015m to 0.009m. Then, from 0.001 to 0.005 cubic metres per second of injection rate, the wellbore width increases to 0.011m. Finally at injection rate of 0.01 cubic metres per second the wellbore width is 0.013m. We can estimate the wellbore width at fracture according to injection rate we use using this graph. For example, we know that for injection rate of 0.0005 cubic metre per second, the wellbore width will be approximately 0.006m.Neglect other factors.

Wellbore width is the width at initial stage of fracturing. It determine the amount of fracturing fluid enter the fracture. By increasing the injection rate the distance the fracturing fluid travel per second also increases. Larger wellbore width will increase the volume of fluid enter the fracture at one time.

Graph below show the effect of different injection rate on wellbore net pressure. When injection rate increases the pressure also increases.

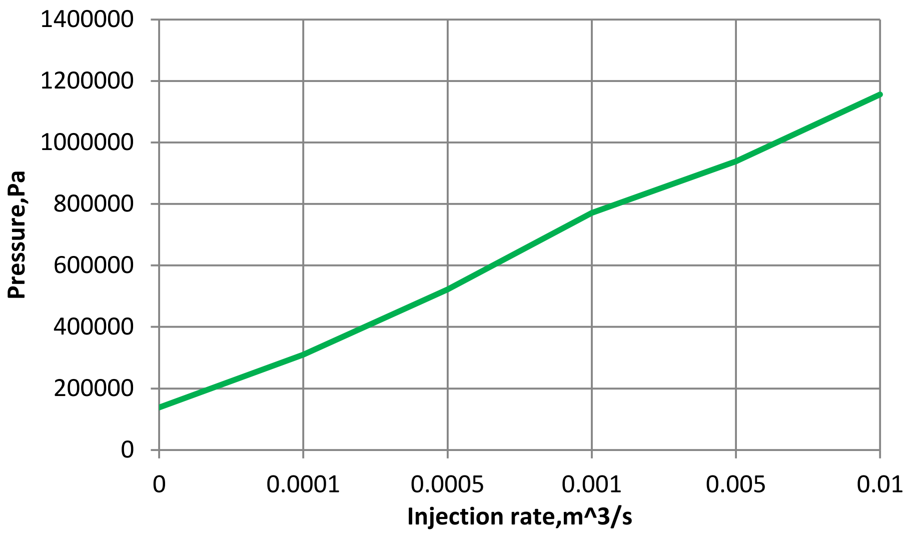


Figure 19: Effect of injection rate on wellbore net pressure

The wellbore net pressure is 150000Pa approximately even when the injection rate is zero. When the injection rate used is 0.0001 cubic metre per second, the pressure rises to 300 000Pa and followed by 500 000Pa at injection rate of 0.0005 cubic metres per second. The pressure at 0.001 cubic metre per second injection rate is almost 800 000Pa, but there is a sharp changes of pressure for the next injection rate this is because different in input data. Hence, the pressure at 0.005 cubic metre per second is only 900 00Pa which also only 100 000Pa difference from 0.001cubic metre per second injection rate. Finally, at 0.01 cubic metre per second the pressure is 1 150 000Pa.

The pressure increases as injection rate increases because pressure is influenced by wellbore width. Wellbore width increases as injection rate increases according to the model. Thus, increasing injection rate will increase the wellbore width and the pressure will also increase.

In conclusion, higher injection rate will give larger fracture with greater width and length but it is not most efficient. This is because higher injection rate will cause higher treating pressure and surface pressure. Based on literature, we know that optimum injection rate depend on optimum injection time.

4.2.3 Injection time of fluid

The input values of injection time insert into mathematical code is tabulated below.

Leak off Coefficient, ⟦m/s⟧^(1/2)	Spurt loss, m	Height, m	Plane strain modulus, Pa	Viscosity, Pa.s	Injection rate,m^3/s	Time,s
0.00000984	0	51.8	62933333330	0.2	0.0662	10500
0.00000984	0	51.8	62933333330	0.2	0.0662	14000
0.00000984	0	51.8	62933333330	0.2	0.0662	17500
0.00000984	0	51.8	62933333330	0.2	0.0662	19000
0.00000984	0	51.8	62933333330	0.2	0.0662	22750
0.00000984	0	51.8	62933333330	0.2	0.0662	25000

Table 3: Input value of injection time

From the graph below, we can see the effect of injection time on average width of fracture. There is significant change in the average width as injection time increases with fixed injection rate that is 0.0662 cubic metres per second.

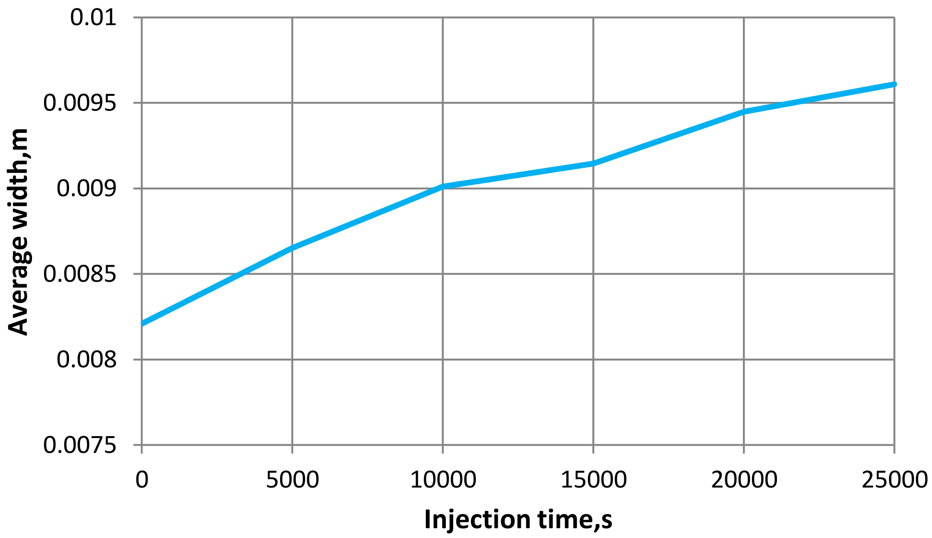


Figure 20: Effect of injection time on average width

When injection time is 0 second the average width is 0.00825 metre which then increases to 0.0087m at 5000s. At 10 000s, the average width increases to 0.009m. From 10000s to 15000s, there are not many changes in average width. The average width only increases 0.0002m from 0.009m, which is 0.0092m. Then, the average width with injection time of 20000s is 0.0094m and followed by 0.0096m

25000s of injection time. The rises in average width from each 5000s injection time decreases as injection increases.

The average width increases when injection time increases because the width along the fracture increases as pressure induced more force on the wall as time increases. Thus, it will increase the average width along the fracture.

This graph shows the effect of injection time on fracture length. The fracture length increases as the injection time increases.

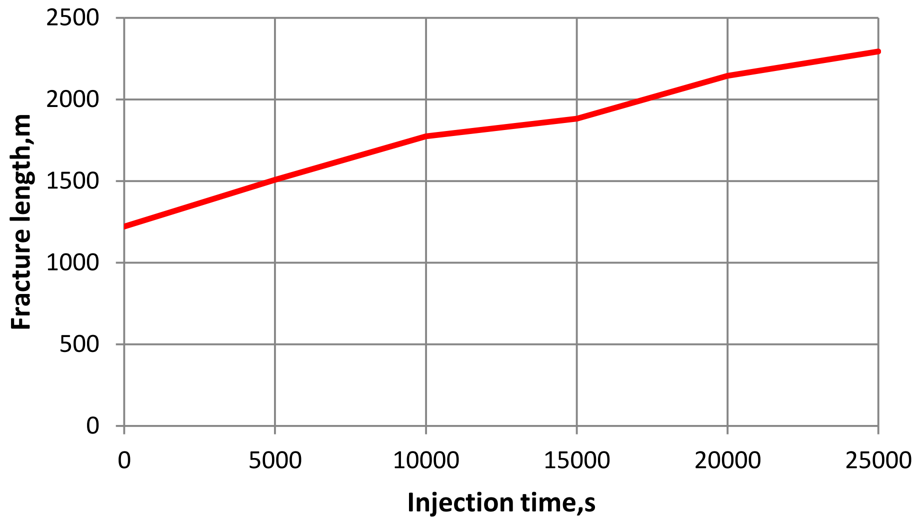


Figure 21: Effect of injection time on fracture length

When injection time is zero, the initial fracture length is 1300metres. As the injection time used increased to 5000 second, the fracture length also increases to 1500m. Besides that, at injection time of 10000s, the fracture length is 1750m and when injection time is 15000s the fracture length increased to 1900m. Then, at injection time of 20000s, the fracture length 2200m and at 25000s the fracture length just increased extra 100m that is 2300m. Similar as average width, after a certain range of injection time the fracture length does not change much.

From the graph, we can understand that the trend of the fracture length graph is same as the average width graph. This is because injection time is a parameter only used to calculate fracture length while fracture length is one of the parameter used to calculate the average width. This is why the average width graph trend is similar to fracture length but it still produces different individual value.

From this graph, we can know the effect difference injection time on wellbore width with fixed injection rate and viscosity of fracturing fluid.

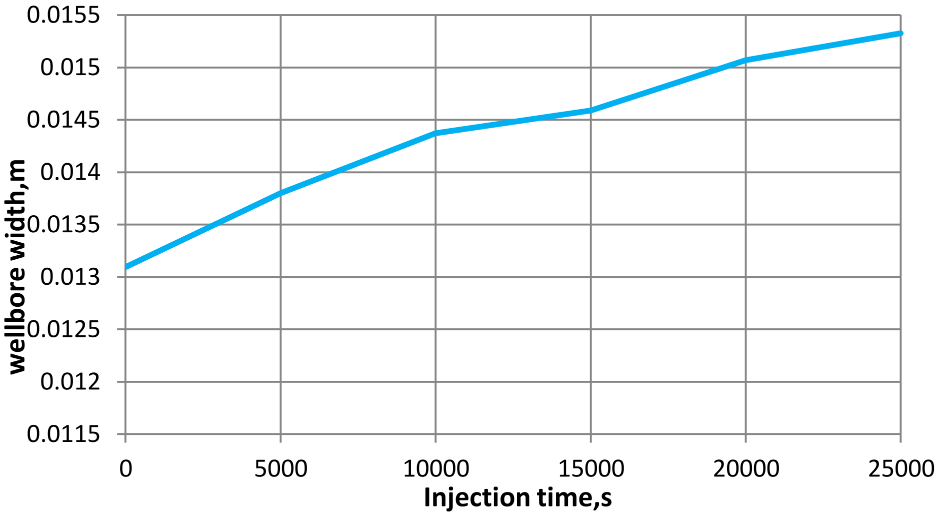


Figure 22: Effect of injection time on wellbore width

Wellbore width increases as injection time increases. The initial wellbore width is 0.0131m when injection time is 0 second. There is a steady went up from 0s to 10000s injection time based on the graph trend. The wellbore width at that period is 0.0138m to 0.0144m. Then, at 15000s of injection time the wellbore width is 0.0146m and followed by 0.0151m at 20000s of injection time. At the end of 25000s injection time, the wellbore width is 0.0154m.

Similar to average width the injection time does not influence wellbore width calculation directly. Injection time influenced the fracture length which is one of the parameter in wellbore width calculation. With other value of parameters is kept constant the trend of wellbore width increment does not change much. After a certain injection time, the effect of injection time in fracture geometry reduces with constant injection rate.

The graph below shows the effect of injection time on wellbore net pressure of the fracture. As the injection time increases the net pressure also increases steadily.

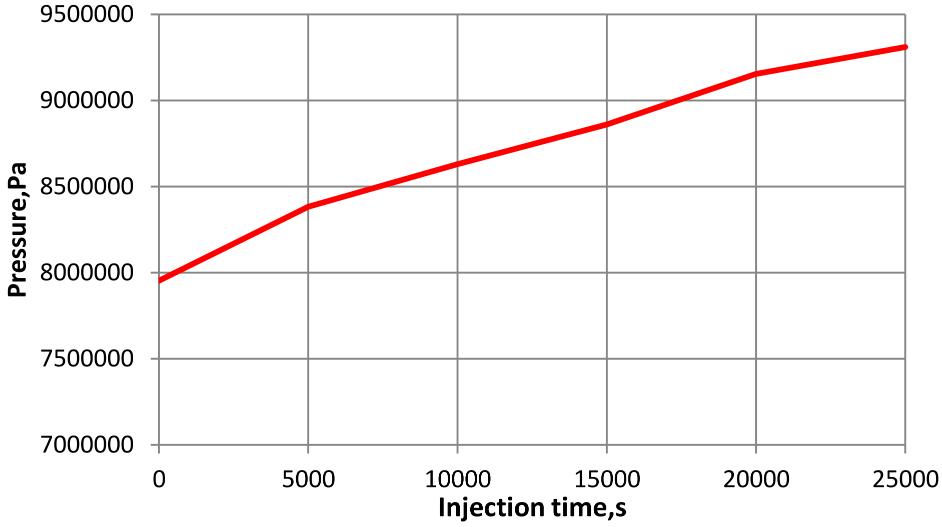


Figure 23: Effect of injection time on wellbore net pressure

The initial pressure at injection time of 0s is 7950000Pa. As the injection time used increases to 5000s, the pressure also increases to 8400000Pa and 8550000Pa at injection time of 10000s. Then, the pressure at injection time of 15000s is 8800000Pa and followed by 9200000Pa when injection time is 20000s. Finally when the injection time is increased to 25000s the pressure also increases to 9350000Pa.

The calculation of pressure for fracture is directly influenced by wellbore width so when the wellbore width increases the as injection time increases the pressure also increases. If injection time increases the pressure exist at wellbore also increases which will create big wellbore width.

In conclusion, to create best fracture geometry optimum injection time is required. Any prolonged injection after injection time will induce unnecessary fracture growth and thus, will add treatment cost.

CHAPTER 5: CONCLUSION AND RECOMMENDATION

As a conclusion, this project is important because it give a exposure and knowledge about newly developing field in oil and gas industry. Hydraulic fracturing in shale gas reservoir is one of the techniques growing rapidly in big countries. Analyse a model with various parameters explained the importance of parameters in a hydraulic fracture geometry.

The project is within capability of a final year student to be executed with the help and guidance from the supervisor and the coordinator. The time frame is also feasible and the project can be completed within the time allocated. It is hoped that the acquiring of equipment and materials needed for the experiment runs smoothly for the accomplishment of this project at the end.

The time frame, source for research, knowledge gained and equipments is the important factor should be considered for final year project. Besides that, there is no specific paper or information about specific formation makes it difficult to find reference. Journals and research paper should be more about specific topic rather than general.

Last but not least, the PKN-C model is verified to determine the hydraulic fracture geometry in shale gas reservoir. The analysis of the outcomes using the parameters is also relevant to phenomena of physics. The results are summarized below:

- Average width, wellbore width and pressure of fracture increases when viscosity of fracturing fluid increases while fracture length decreases as viscosity of fluid increases. Viscosity influenced mobility of fracturing fluid and the proppant concentration it transport down the hole.
- Average width, fracture length, wellbore width and pressure increases as injection rate of fluid increases. This is because injection rate influenced the formation of fracture in total. Optimum injection rate depend on optimum injection time.

- Average width, fracture length, wellbore width and pressure increases as injection time increases. Injection time does not influence average width, wellbore width and pressure directly. It only influenced fracture length directly so all the other outcome is based on fracture length behaviour.

Recommendations for future work are:

- Use of different shale reservoir data for different place to analyse the model. This is because shale rock characteristics vary from one place to another place.
- This study should be also carried out in 3D model to compare the result with 2D model. From this comparison, the percentage of accuracy of 2D model verification can be determined.

REFERENCE

[1] Nolte, K. G. (1979). Determination of fracture parameters from fracture pressure decline(SPE-8341). Amoco Production Co.

[2] Hydraulic Fracture Geometry: Fracture Containment in Layered Formations. (1982). Society of Petroleum Engineers Journal. doi:10.2118/9261-PA

[3] Gidley, J. L., & Society of Petroleum Engineers (U.S.) (1989). Recent advances in hydraulic fracturing. Richardson, TX: Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers.

[4] Valkó, P., & Economides, M. J. (1995). Hydraulic fracture mechanics. Chichester: Wiley.

[5] Smith, M. B., & Shlyapobersky, J. W. (1996). Basic of hydraulic fracturing. In Reservoir stimulation (pp. 5.1-5.28).

[6] Rahman, M. M., Rahman, M. K., & Rahman, S. S. (2001). An integrated model for multiobjective design optimization of hydraulic fracturing

[7] Merriman, R. J., Highley, D. E., & Cameron, D. G. (2003). Definition and characteristics of very-fine grained sedimentary rocks: clay,mudstone, shale and slate.

[8] Definition and characteristics of very-fine grained sedimentary rocks : clay, mudstone, shale and slate: British Geological Survey report CR/03/281N. (2003).

[9] Detoumay, E. (2004). Propagation Regimes of Fluid-Driven Fractures in Impermeable Rocks.

[10] Bennett, L., Calvez, J. L., Sarver, D. R., & Tanner, K. (2005). The source for hydraulic fracture characterization.

[11] Adachi, J., Siebrits, E., Peirce, A., & Desroches, J. (2007). Computer simulation of hydraulic fractures.

[12] Jenkins, C., & II, C. B. (2008). Coalbed and Shale-Gas Reservoirs. Journal of Petroleum Technology. doi:10.2118/103514-MS

[13] Cherny, S., Chirkov, D., Lapin, V., Muranov, A., Banikov, D., Miller, M., Willberg, D. (2009). Two-dimensional modeling of the near-wellbore fracture tortuosity effect.

[14] Kim, Gun-Ho. (2010). Interpretation of hydraulic fracturing pressure in low-permeability gas reservoirs. University Park, Pa.: Pennsylvania State University.

[15] Xiang, J. (2011). A PKN hydraulic fracture model study and formation permeability determination.

[16] Zhang, X., Jeffrey, R. G., Bunger, A. P., & Thiercelin, M. (2011). Initiation and growth of a hydraulic fracture from a circular wellbore. International Journal of Rock Mechanics and Mining Sciences. doi:10.1016/j.ijrmms.2011.06.005

[17] Treadgold, G., B. Campbell, B. McLain, S. Sinclair, and D. Nicklin, 2011, Eagle Ford shale prospecting with 3D seismic data within a tectonic and depositional system framework.

[18] Freeman, C. M., Moridis, G. J., & Blasingame, T. A. (2011). A Numerical Study of Microscale Flow Behavior in Tight Gas and Shale Gas Reservoir Systems.

[19] Olorode, Olufemi Morounfopefoluwa. (2012). Numerical modeling of fractured shale-gas and tight-gas reservoirs using unstructured grids. College Station, Tex: Texas A&M University.

[20] Chopra, S., Sharma, R. K., Keay, J., & Marfurt, K. J. (2012). Shale gas reservoir characterization workflows.

[21] Montgomery, C. (2013). Fracturing fluid. NSI Technologies, Tulsa, Oklahoma, USA

[22] Gomaa, A. M., Qu, Q., Maharidge, R., Nelson, S., & Reed, T. (2014). New insight into hydraulic fracturing of shale formations.

APPENDIX

```
File   Edit   Format   View   Help
% ask user for input
% initial
% CL = 9.84*10e-6;
% Sp = 0;
% hf = 51.8;
% dE = 6.13 * 10e10;
% mu = 0.2;
% i = 0.0662;
% xf = 1000;

function [EW, xfsolved , W_WO, P_nw] = myfun_xf(CL,Sp,hf,dE,mu,i,t)

n = size(CL,1);
xfsolved = zeros(n,1);
EW = zeros(n,1);

for k = 1:n
syms xf EW

[xfsolved(k), EW(k)] = solve(xf == ((EW+2*Sp(k))*i(k))/(4*CL(k)^2*pi*hf(k))*...
           (exp((2*CL(k)*sqrt(pi*t(k))/(EW+2*Sp(k)))^2 )*erfc((2*CL(k)*sqrt(pi*t(k)))/(EW+2*Sp(k)))+...
           (2*(2*CL(k)*sqrt(pi*t(k)))/(EW+2*Sp(k)))/(sqrt(pi))-1),EW == 2.05*((mu(k).*i(k).*xf)./dE(k)).^0.25, xf, EW);

EW = 2.05*((mu.*i.*xfsolved)./dE).^0.25;
W_WO = 3.27*((mu.*i.*xfsolved)./dE).^0.25;
P_nw = (dE./(2.*hf)).*W_WO;
end



figure;
subplot(2,2,1)
plot(xfsolved,dE)
title('Youngs modulus vs Length')
subplot(2,2,2)
plot(EW,dE)
title('Youngs modulus vs Width')
subplot(2,2,3)
plot(P_nw,dE)
title('Youngs modulus vs Net Pressure')
subplot(2,2,4)
plot(P_nw,EW)
title('Width vs Net Pressure')

end
```

Figure 1- Mathematical code (Stimulation mode)

```
File   Edit   Format   View   Help
% ask user for input
% initial
% for example
% CL = 9.84*10e-6;
% Sp = 0;
% hf = 51.8;
% dE = 6.13 * 10e10;
% mu = 0.2;
% i = 0.0662;
% xf = 1000;

% read from exel file
filename = 'myExamplexf.xlsx';
% CL = first col
% Sp = second col
% hf = third col
% dE = forth col
% mu = fifth col
% i = sixth col
% xf = seventh col
data = xlsread(filename);

[EW, xfsolved , W_W0, P_nw] = myfun_xf(data(:,1),data(:,2),data(:,3),data(:,4),data(:,5),data(:,6),data(:,7));

results = [EW,xfsolved,W_W0,P_nw];
Names_var =  {'EW'; 'xf'; 'W_W0';'P_nw'};

%write data in csv format
fid = fopen('Results_xf.csv','w');
fprintf(fid, [Names_var{1} sprintf(',%s',Names_var{2:end}) '\n']);
fclose(fid)
dlmwrite('Results_xf.csv', results, '-append', ...
         'delimiter', ',','precision','%.10f');
```

Figure 2- Mathematical code which will analyze stimulation mode

Type of models	Parameters
PKN-C	
$\overline{w} = \frac{\pi}{5} w_{w,0} = 2.05 \left(\frac{\mu i \, x_f}{E'}\right)^{1/4}$	- $\overline{w}$: Average width, m - μ : Viscosity of the fluid, Pa.s - x_f : Fracture length, m - i : Injection rate, $m^3.s^{-1}$ - E' : Plane strain modulus, Pa
$x_f = \frac{(\overline{w} + 2S_p)i}{4C_L^2 \pi h_f} [\exp(\beta^2) \operatorname{erfc}(\beta) + \frac{2\beta}{\sqrt{\pi}} - 1]$ Where $\beta = \frac{2C_L\sqrt{\pi t}}{\overline{w} + 2S_p}$	- x_f : Fracture length, m - $\overline{w}$: Average width, m - S_p : Spurt loss coefficient, m - i : Injection rate, $m^3.s^{-1}$ - C_L : Leak off coefficient, $m.s^{-1/2}$ - h_f : Constant fracture height, m - β : Auxiliary variable for Carter equation II, dimensionless - t : Time, s
$p_{n,w} = \frac{E'}{2h_f} w_{w,0}$	- $p_{n,w}$: Wellbore net pressure, Pa - E' : Plane strain modulus, Pa - h_f : Constant fracture height, m - $w_{w,0}$: Maximum fracture width at wellbore, m
KGD-C	
$\overline{w} = \frac{\pi}{5} w_{w,0} = 2.05 \left(\frac{\mu i \, x_f}{E'}\right)^{1/4}$	- $\overline{w}$: Average width, m - μ : Viscosity of the fluid, Pa.s - x_f : Fracture length, m - i : Injection rate, $m^3.s^{-1}$ - E' : Plane strain modulus, Pa

$x_f = \frac{(\bar{w} + 2S_p)i}{4C_L^2 \pi h_f}$ [exp (β^2) erfc (β) + $\frac{2\beta}{\sqrt{\pi}}$ -1] Where $\beta = \frac{2C_L\sqrt{\pi t}}{\bar{w}+2S_p}$	-x_f : Fracture length, m -$\bar{w}$: Average width, m -S_p : Spurt loss coefficient, m - i : Injection rate, $m^3.s^{-1}$ -C_L : Leak off coefficient, $m.s^{-1/2}$ -h_f : Constant fracture height, m -β : Auxiliary variable for Carter equation II, dimensionless - t : time, s
PKN-N & KGD-N	
$\bar{w} = \frac{\pi}{5} w_{w,0} = 2.05 \left(\frac{\mu i\, x_f}{E'}\right)^{1/4}$	-$\bar{w}$: Average width, m -μ : Viscosity of the fluid, Pa.s -x_f : Fracture length, m -i : Injection rate, $m^3.s^{-1}$ -E' : Plane strain modulus, Pa
Find t : $\frac{it}{h_f x_f} = \bar{w} + C_L\sqrt{t}\,[\frac{8}{3}\eta + (1-\eta)\pi]$ Where $\eta = h_f x_f \bar{w}/it$	- i : Injection rate, $m^3.s^{-1}$ - t : time, s -h_f : Constant fracture height, m -x_f : Fracture length, m -$\bar{w}$: Average width, m -C_L : Leak off coefficient, $m.s^{-1/2}$ -η : Fluid efficiency, dimensionless
$p_{n,w} = \frac{E'}{2h_f} w_{w,0}$	-$p_{n,w}$: Wellbore net pressure, Pa -E' : Plane strain modulus, Pa -h_f : Constant fracture height, m -$w_{w,0}$: Maximum fracture width at wellbore, m

PKN-α & KGD- α	
$x_f = \dfrac{\frac{it}{h_f}}{\overline{w}+2S_p+2C_L\sqrt{t}[\frac{\alpha\sqrt{\pi}\,\Gamma(\alpha)}{\Gamma\left(\frac{3}{2}+\alpha\right)}]}$ - to calculate the length at any time during injection without referring to the width or efficiency at the end of pumping	- i : Injection rate, $m^3.s^{-1}$ - t : time, s -h_f : Constant fracture height, m -x_f : Fracture length, m -$\overline{w}$: Average width, m -S_p : Spurt loss coefficient, m -C_L : Leak off coefficient, $m.s^{-1/2}$ - α :Exponent of fracture length growth, dimensionless - Γ :Foam quality, dimensionless ratio

Table 1- Type of 2D models and parameters